https://www.linkedin.com/feed/update/urn:li:activity:6554625843396775936

4348. Proposed by Marius Drăgan.

Let $p \in [0,1]$. Then for each $n > 1$, prove that

$(1-p)^n + p^n \geq (2p^2 - 2p + 1)^n + (2p - 2p^2)^n.$

Solution by Arkady Alt, San Jose, California, USA.

Note that $(1-p)^n + p^n \geq (2p^2 - 2p + 1)^n + (2p - 2p^2)^n \iff$

$(2-2p)^n + (2p)^n \geq (4p^2 - 4p + 2)^n + (4p - 4p^2)^n \iff$

(**1**) $(1+x)^n + (1-x)^n \geq (1+x^2)^n + (1-x^2)^n$, where $x := 1 - 2p \in [-1,1]$.

$(1+x)^n + (1-x)^n = 2\sum_{0 \leq k, 2k \leq n} \binom{n}{2k} x^{2k}$,

$(1+x^2)^n + (1-x)^n = 2\sum_{0 \leq k, 2k \leq n} \binom{n}{2k} x^{4k}$ and $x^{2k} \geq x^{4k} \iff 1 \geq |x|, k = 1, 2, \ldots$

implies (**1**).